SADLER UNIT 4 CHAPTER 11

EXERCISE 11A

Q1 $v = 6t\sqrt{16+t^2}$

a) $a(t) = 6t\left(\frac{1}{2}(16+t^2)^{-\frac{1}{2}}\right).2t + 6\sqrt{16+t^2}$

$= \dfrac{6t^2}{\sqrt{16+t^2}} + 6\sqrt{16+t^2}$

$a(0) = 6\sqrt{16}$

$= \underline{\underline{24\ m/s^2}}$

b) $x(t) = \int 6t\sqrt{16+t^2}\,dt$

Let $u = 16+t^2$

$\dfrac{du}{dt} = 2t$

$dt = \dfrac{1}{2t}du$

$x(t) = \int 3\sqrt{u}\,du$

$= \dfrac{3u^{3/2}}{\left(\frac{3}{2}\right)} + c$

$= 2\sqrt{(16+t^2)^3} + c$

$8 = 2\sqrt{16}^3 + c$

$8 = 2(64) + c$

$c = -120$

$x(t) = 2\sqrt{(16+t^2)^3} - 120$

$x(3) = 2\sqrt{25}^3 - 120$

$= \underline{\underline{130\ m}}$

Q2 $a = \dfrac{6t(t+1)^2}{5}$

$v(t) = \int \frac{6}{5}t(t+1)^2\,dt$

Let $u = t+1 \Rightarrow t = u-1$

$du = dt$

$v(t) = \int \frac{6}{5}(u-1)u^2\,du$

$= \frac{6}{5}\int u^3 - u^2\,du$

$= \frac{6}{5}\left(\frac{u^4}{4} - \frac{u^3}{3}\right) + c$

$= \dfrac{3(t+1)^4}{10} - \dfrac{2(t+1)^3}{5} + c$

$v(1) = 2$

$2 = 0.3(16) - 0.4(8) + c$

$c = \underline{\underline{0.4}}$

$\therefore v(t) = \dfrac{3(t+1)^4 - 4(t+1)^3 + 4}{10}$

$\therefore v(0) = \dfrac{3-4+4}{10}$

$= \underline{\underline{0.3\ m/s}}$

Q3 $x = 5 + 2\cos t$

$v(t) = -2\sin t$

$a(t) = -2\cos(t)$.

a) $v\left(\frac{\pi}{6}\right) = -2\sin\left(\frac{\pi}{6}\right)$

$= -1\ m/s$.

b) $a\left(\frac{\pi}{2}\right) = 0\ m/s^2$

Q4. $v = 4\sin 2t$

$a(t) = 8\cos(2t)$

$x(t) = \int 4\sin(2t)\,dt$

$= -2\cos(2t) + c$

a) $a\left(\frac{\pi}{6}\right) = 8\cos\left(\frac{\pi}{3}\right)$

$= 4\ m/s^2$.

b) $3 = -2\cos(0) + c$

$c = 5$

$\therefore x(t) = -2\cos(2t) + 5$

$x\left(\frac{\pi}{2}\right) = -2\cos(\pi) + 5$

$= \underline{\underline{7\ m}}$

Q5 $a(t) = 4\sin t\cos t = 2\sin(2t)$

$x(0) = 5 \qquad v(0) = 3$.

$a(t) = 2\sin(2t)$

a) $V(t) = \int 2\sin(2t)\, dt$

$= -\cos(2t) + c$

$3 = -\cos(0) + c$

$3 = -1 + c$

$\underline{\underline{c = 4}}$

$\therefore V(t) = -\cos(2t) + 4$

$V(\frac{\pi}{3}) = -\cos(\frac{2\pi}{3}) + 4$

$= -(-\frac{1}{2}) + 4$

$= \underline{\underline{4.5 \text{ m/s}}}$

b) $x(t) = \int -\cos(2t) + 4\, dt$

$= -\frac{1}{2}\sin(2t) + 4t + c$

$5 = -\frac{1}{2}(0) + 0 + c$

$\underline{\underline{c = 5}}$

$\therefore x(t) = -\frac{1}{2}\sin(2t) + 4t + 5$

$x(\frac{\pi}{3}) = -\frac{1}{2}\sin(\frac{2\pi}{3}) + \frac{4\pi}{3} + 5$

$= -\frac{1}{2}(\frac{\sqrt{3}}{2}) + \frac{4\pi}{3} + 5$

$= \underline{\underline{\frac{4\pi}{3} + 5 - \frac{\sqrt{3}}{4} \text{ m}}}$

Q6. $v = 5 + x^2$

$\frac{dv}{dt} = \frac{d}{dt}(5 + x^2)$

$a(t) = 2x \frac{dx}{dt}$

$a(t) = 2x(5 + x^2)$

$\Rightarrow a(x) = 2x(5 + x^2)$

$a(1) = 2(5+1)$

$= \underline{\underline{12 \text{ m/s}^2}}$

Q7. $a = 3x^2 + 1$

$\frac{dv}{dt} = 3x^2 + 1$

$\frac{dv}{dx} \times \frac{dx}{dt} = 3x^2 + 1$

$\int v\, dv = \int 3x^2 + 1\, dx$

$\frac{v^2}{2} = \frac{3x^3}{3} + x + c$

$\frac{v^2}{2} = x^3 + x + c$

$\frac{4}{2} = c \Rightarrow c = 2$

$\frac{v^2}{2} = x^3 + x + 2$ //

$V(3) \Rightarrow$

$\frac{v^2}{2} = 27 + 3 + 2$

$v^2 = 64$

$v = \pm 8 \quad (v > 0)$

$\therefore \underline{\underline{v = 8 \text{ m/s}}}$

Q8 $a = v^2$

$\frac{dv}{dt} = v^2$

$\int \frac{1}{v^2}\, dv = \int 1\, dt$

$-\frac{1}{v} = t + c$

When $t = 2$, $v = 0.1$

$-\frac{1}{0.1} = 2 + c$

$-10 = 2 + c$

$\underline{\underline{c = -12}}$

$\therefore -\frac{1}{v} = t - 12$

$v = -\frac{1}{t-12}$

$x = \int -\frac{1}{t-12}\, dt$

$= -\ln|t-12| + c$

$x = \ln\left|\frac{c}{t-12}\right|$

$0 = \ln\left|\frac{c}{2-12}\right|$

$1 = \frac{c}{-10}$

$c = -10$ //

$x = \ln\left|\frac{-10}{t-12}\right|$

a) $v(10) = -\frac{1}{10-12}$

$= \frac{1}{2} = \underline{\underline{0.5 \text{ m/s}}}$

b) $x = 2$,

$z = \ln\left|\frac{-10}{t-12}\right|$

$e^2 = \frac{-10}{t-12}$

$t - 12 = \frac{-10}{e^2}$

$t = -\frac{10}{e^2} + 12$

$\therefore v = -\frac{1}{-\frac{10}{e^2}+12-12}$

$= \frac{-e^2}{-10}$

$= \underline{\underline{0.1e^2}} \text{ m/s}$.

Q9. $x = \frac{t+1}{2t+3}$, $t \geq 0$.

a) $v(t) = \frac{(2t+3)(1) - 2(t+1)}{(2t+3)^2}$

$= \frac{2t+3-2t-2}{(2t+3)^2}$

$v(t) = \frac{1}{(2t+3)^2}$ m/s

$a(t) = \frac{(2t+3)^2(0) - 4(2t+3)}{(2t+3)^4}$

$= \underline{\underline{-\frac{4}{(2t+3)^3}}}$ m/s^2

b) $x(1) = 0.4$ m $a(1) = -0.032$ m/s^2

$v(1) = 0.04$ m/s.

Q10. $h = 42 + 29t - 5t^2$, $t \geq 0$.

$0 = 42 + 29t - 5t^2$

$t = \frac{-29 \pm \sqrt{29^2 - 4(-5)(42)}}{-10}$

$= \frac{-29 \pm \sqrt{1681}}{-10}$

$= \frac{-29 \pm 41}{-10}$

$t = -1.2$ or $\underline{\underline{7}}$ secs.

(reject)

$\frac{dh}{dt} = 29 - 10t$

$\left.\frac{dh}{dt}\right|_{t=7} = 29 - 70$

$= -41$

$\therefore |-41| = \underline{\underline{41 \text{ m/s}}}$

Q11. $x = t(16-t)$

$x = 16t - t^2$

a) $v = 16 - 2t$

$|v(20)| = |16 - 40|$

$= \underline{\underline{24 \text{ m/s}}}$.

b) $0 = 16 - 2t$ $x = 8(16-8)$

$\underline{\underline{t = 8 \text{ secs}}}$ $= \underline{\underline{64 \text{ m}}}$

c) $D = \int_1^5 16 - 2t \, dt$

$= \left[16t - t^2\right]_1^5$

$= 80 - 25 - 16 + 1$

$= \underline{\underline{40 \text{ m}}}$

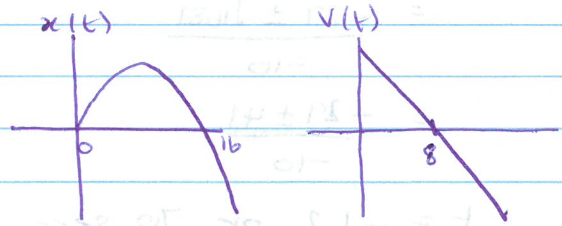
d) $D = \int_5^8 16 - 2t\,dt - \int_8^{10} 16 - 2t\,dt$

$= \left[16t - t^2\right]_5^8 - \left[16t - t^2\right]_8^{10}$

$$= \left[16t - t^2\right]_5^8 - \left[16t - t^2\right]_8^{10}$$

$$= 16(8) - 64 - 80 + 25 - \left[160 - 100 - 16(8) + 64\right]$$

$$= 13\,m$$



Q12. $v(0) = 35$

$a(t) = 6(t-4)$

$v = \frac{6t^2}{2} - 24t + c$

$\therefore v = 3t^2 - 24t + 35$

$x = t^3 - 12t^2 + 35t + 0$

$0 = t(t^2 - 12t + 35)$

$0 = t(t-7)(t-5)$

$\therefore$ At O at $t = 0$

$\boxed{t = 5}$

$t = 7$

$v(5) = 3(25) - 24(5) + 35$

$= -10\,m/s$

Q13. $x(0) = 0$

$v = 2\sin(2t)$

a) Max $v = 2\,m/s$.

b) $a(t) = 4\cos(2t)\ m/s^2$

c) Max $a = 4\,m/s^2$

d) $x(t) = -\cos(2t) + c$

$0 = -\cos(0) + c$

$c = 1$

$\therefore x(t) = -\cos(2t) + 1$

e) Max $x = 1 + 1$

$= 2\,m$

Q14 $v = 3x + 2$

a) $a = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$

$= \frac{d}{dx}\left(\frac{1}{2}(3x+2)^2\right)$

$= 3(3x+2)$

$= (9x + 6)\,m/s^2$

b) $v(4) = 14\,m/s$

$a(4) = 9(4) + 6$

$= 42\,m/s^2$

Q15 $x(0) = 0$, $v(0) = 4$

$a = -(1 + v^2)$

$\frac{dv}{dt} = -(1 + v^2)$

$\int \frac{1}{1+v^2}\,dv = \int -1\,dt$

Let $v = \tan\theta$

$\frac{dv}{d\theta} = \sec^2\theta$.

$\int \frac{1}{1+\tan^2\theta}\sec^2\theta\,d\theta = \int -1\,dt$

$\int 1\,d\theta = \int -1\,dt$

$\theta = -t + c$

When $v = 4$, $t = 0$ and $\theta = \tan^{-1}(4)$.

$\tan^{-1}(4) = c$

$\therefore \theta = -t + \tan^{-1}(4)$.

When $v=1$, $\theta=\frac{\pi}{4}$

$$\frac{\pi}{4} = -t + \tan^{-1}(4)$$
$$t = \tan^{-1}(4) - \frac{\pi}{4}$$

$$t = 0.5404195$$

$$\tan^{-1}(v) = -t + \tan^{-1}(4)$$
$$v = \tan(-t + \tan^{-1}(4))$$

$$D = \int_0^{0.5404195} |\tan(-t + \tan^{-1}(4))| \, dt$$

$$= \underline{\underline{1.4166 \text{ m.}}}$$

Hmmm... contradictory to solutions.

$$\frac{dv}{dt} = -(1+v^2)$$

$$\frac{dv}{dx}\frac{dx}{dt} = -(1+v^2)$$

$$\frac{dv}{dx} v = -(1+v^2)$$

$$\int \frac{v}{(1+v^2)} dv = \int -1 \, dx$$

$$\frac{1}{2}\ln|1+v^2| = -x + c_1$$
$$\ln(1+v^2) = -2x + c_2$$
$$1+v^2 = e^{-2x} \cdot e^{c_2}$$
$$1+v^2 = c_3 e^{-2x}$$
$$v^2 = c_3 e^{-2x} - 1$$

When $x=0$, $v=4$.

$$16 = c_3 - 1$$
$$c_3 = 17$$

$$\therefore v^2 = 17e^{-2x} - 1$$

When $v=1$,

$$1+1 = 17e^{-2x}$$
$$\frac{2}{17} = e^{-2x}$$
$$\ln\left(\frac{2}{17}\right) = -2x.$$
$$x = \frac{\ln\left(\frac{2}{17}\right)}{-2} \quad \text{or } \frac{1}{2}\ln\left(\frac{17}{2}\right)$$
$$= \underline{\underline{1.07 \text{ M}}}$$

EXERCISE 11B

Q1 a) $|a| = 5\text{m}, \ P = \pi \text{ s}$

b) $|a| = 4\text{m}, \ P = \frac{2\pi}{5} \text{ s.}$

c) $|a| = 2\text{m}, \ P = \frac{\pi}{2} \text{ s.}$

Q2 a) If $\ddot{x} = -4x$

then $-k^2 = -4$

$k = 2$

$\Rightarrow P = \pi$ secs

b) If $\ddot{x} = -x$

then $-k^2 = -1$

$k = 1$

$\therefore P = 2\pi$ secs

c) If $\ddot{x} = -25x$

then $-k^2 = -25$

$k = 5$

$\therefore P = \frac{2\pi}{5}$ secs

Q3 Let $x = A\sin(kt)$

a) $x = \sin(\frac{1}{2}t)$

b) $x = -\sin(\frac{1}{2}t)$

c) $x = 3\sin(2t)$

d) $x = -\frac{1}{2}\sin(\pi t)$

Q4. Let $x = A\cos(kt)$

a) $x = 2\cos(2t)$

b) $x = 1.5\cos(4t)$

c) $x = \frac{1}{2}\cos(4\pi t)$.

Q5. Let $x = A\sin(kt)$

a) $x = 2.5\sin(2t)$

or $x = -2.5\sin(2t)$

b) $v(t) = \pm 5\cos(2t)$.

$v(\frac{\pi}{6}) = \pm 5(\frac{1}{2})$

$|v(\frac{\pi}{6})| = 2.5 \text{ m/s}$

Q6. a) $x = 5\cos(5t) + 3\sin(5t)$

$A = \sqrt{5^2 + 3^2}$

$= \sqrt{34}$ m.

$P = \frac{2\pi}{5}$ secs

b) $x = 3\cos(2t) + 7\sin(2t)$

$A = \sqrt{9+49}$

$= \sqrt{58}$ m

$P = \frac{2\pi}{2} = \pi$ secs

Q7 $x = 4\sin(\frac{\pi}{10}t)$

a) $v(t) = \frac{4\pi}{10}\cos(\frac{\pi}{10}t)$

$a(t) = -\frac{4\pi^2}{100}\sin(\frac{\pi}{10}t)$

$\ddot{x} = -\frac{\pi^2}{100}x$

$\therefore$ Simple harmonic motion

b) $A = 4$ m, $P = \frac{2\pi}{(\frac{\pi}{10})} = 20$ secs

c) $D = \int_0^2 \left|\frac{4\pi}{10}\cos(\frac{\pi}{10}t)\right| dt$

$= \left[4\sin(\frac{\pi}{10}t)\right]_0^2$

$= 4\sin(\frac{\pi}{5})$

$= 2.35$ m

Q8. $x = 2\sin(\frac{\pi}{3}t)$

a) $v = \frac{2\pi}{3}\cos(\frac{\pi}{3}t)$

$a = -\frac{2\pi^2}{9}\sin(\frac{\pi}{3}t)$

$\therefore \ddot{x} = -\frac{\pi^2}{9}x$

$\therefore$ Simple harmonic motion.

b) $A = 2\text{m}$

$P = \dfrac{2\pi}{\left(\frac{\pi}{3}\right)}$

$P = 6$ secs

c) $D = \int_0^2 \left|\frac{2\pi}{3}\cos\left(\frac{\pi}{3}t\right)\right| dt$

$= \int_0^{1.5} \frac{2\pi}{3}\cos\left(\frac{\pi}{3}t\right) dt - \int_{1.5}^{2} \frac{2\pi}{3}\cos\left(\frac{\pi}{3}t\right) dt$

$= \left[2\sin\left(\frac{\pi}{3}t\right)\right]_0^{1.5} - \left[2\sin\left(\frac{\pi}{3}t\right)\right]_{1.5}^{2}$

$= 2\sin\frac{\pi}{2} - 0 - \left(2\sin\left(\frac{2\pi}{3}\right) - 2\sin\frac{\pi}{2}\right)$

$= 2 - \sqrt{3} + 2$

$= 4 - \sqrt{3}$ m.

Q9. $x = 3\sin\left(2t + \frac{\pi}{6}\right)$

a) $v = 6\cos\left(2t + \frac{\pi}{6}\right)$

$a = -12\sin\left(2t + \frac{\pi}{6}\right)$

$\ddot{x} = -4x$

$\therefore$ simple harmonic motion

b) $P = \frac{2\pi}{2} = \pi$ secs.

$A = 3$ m.

c) $\int_0^1 \left|6\cos\left(2t + \frac{\pi}{6}\right)\right| dt$

$= 2.76$ m.

Q10. Let $x = a\sin(kt + \alpha)$

a) $x = 4\sin(\pi t + \alpha)$

$v = 4\pi\cos(\pi t + \alpha)$

If $v < 0$ when $t = 0$, then

$\frac{\pi}{2} < \alpha \leq \pi$

$2 = 4\sin(\pi(0) + \alpha)$

$\frac{1}{2} = \sin(\alpha) \Rightarrow \alpha = \frac{5\pi}{6}$

b) $v(t) = 4\pi\cos\left(\pi t + \frac{5\pi}{6}\right)$

$v\left(\frac{1}{6}\right) = 4\pi\cos\left(\frac{\pi}{6} + \frac{5\pi}{6}\right)$

$= 4\pi\cos(\pi)$

$= -4\pi$ m/s

$\therefore \left|v\left(\frac{1}{6}\right)\right| = 4\pi$ m/s

Q11. Let $x = a\sin(kt + \alpha)$

a) $x = 2\sin(5t + \alpha)$

$v = 10\cos(5t + \alpha)$

If $v > 0$ when $t = 0$, then

$0 \leq \alpha < \frac{\pi}{2}$.

$\sqrt{2} = 2\sin(\alpha)$

$\frac{\sqrt{2}}{2} = \sin\alpha$

$\alpha = \frac{\pi}{4}$.

$\therefore x = 2\sin\left(5t + \frac{\pi}{4}\right)$

b) $\max v = |10|$

$= 10$ m/s.

c) $a = -50\sin\left(5t + \frac{\pi}{4}\right)$

$\max a = |-50|$

$= 50\ \text{m/s}^2$

Q12. $\ddot{x} = -4x$

Let $x = A\sin(kt + \alpha)$, $\alpha = 0$.

$x = A\sin(kt)$

$x = 0.6\sin(2t)$

a) $x = 0.6\sin\left(\frac{2\pi}{6}\right)$

$= \frac{3}{5}\left(\frac{\sqrt{3}}{2}\right) = \frac{3\sqrt{3}}{10}$ m

b) $x = 0.6\sin\left(\frac{2\pi}{3}\right)$

$x = \frac{3}{5}\left(\frac{\sqrt{3}}{2}\right)$

$= \frac{3\sqrt{3}}{10}$ m

c) $\pm 0.3 = 0.6\sin(2t)$

$\pm\frac{1}{2} = \sin(2t)$

$2t = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}$

i) $t = \frac{\pi}{12}$ secs

ii) $t = \frac{5\pi}{12}$ secs.

iii) $t = \frac{7\pi}{12}$ secs

Q13. $\ddot{x} = -\pi^2 x$

$t=0,\ x=0,\ v<0.$

$|a| = 3$ and $P = 2$

Let $x = -3\sin(\pi t)$

a) $x\left(\frac{1}{3}\right) = -3\sin\left(\frac{\pi}{3}\right)$

$= -\frac{3\sqrt{3}}{2}$ m

b) $v(t) = -3\pi\cos(\pi t)$

$v\left(\frac{1}{3}\right) = -3\pi\cos\left(\frac{\pi}{3}\right)$

$= -\frac{3\pi}{2}$ m/s.

c) $\left|v\left(\frac{1}{3}\right)\right| = \frac{3\pi}{2}$ m/s

d) $\pm\frac{3\pi}{2} = -3\pi\cos(\pi t)$

$\mp\frac{1}{2} = \cos(\pi t)$

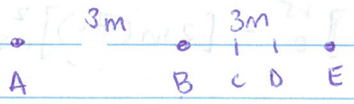
$\mp\frac{1}{2} = \cos(\pi t)$

$\pi t = \frac{\pi}{3}, \frac{2\pi}{3}$

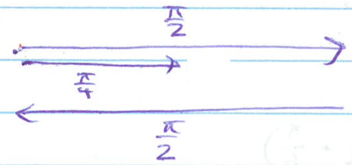
$\therefore t = \frac{1}{3}$ and $\frac{2}{3}$

$\therefore \frac{2}{3}$ seconds

Q14.



$P = \pi$ seconds



Let $x = -3\cos(2t)$ from A.

a) $C \Rightarrow x = 1.$

$1 = -3\cos(2t)$

$2t = 1.9106$

$t = 0.9553$

$t \approx 0.96$ secs

b) $D \Rightarrow x = 2$

$2 = -3\cos(2t)$

$2t = 2.3005$

$t = 1.1503$

$\therefore 1.1503 - 0.9553$

$t \approx 0.19$ secs

c) $DE \Rightarrow t = \frac{\pi}{2}$

$\therefore t = \frac{\pi}{2} - 1.1503$

$t = 0.4205$

$t \approx 0.42$ secs

d) $D \to E \to D$

$t = 0.42 + 0.42$

$t \approx 0.84$ secs.

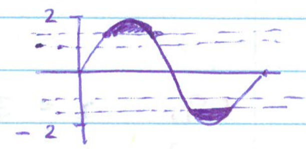
or

$D \to A \to D$

$t = 1.1503 + 1.1503$

$t \approx 2.30$ secs

Q15 $x = 2\sin(4t)$.



$1.5 = 2\sin(4t)$

$\sin(4t) = \frac{3}{4}$

$t = 0.2120,$

and

$t = 0.5734$

are first 2 times

$\therefore 0.5734 - 0.2120$

$= 0.3614$ secs

0.3614×2

$= 0.7227$

≈ 0.72 secs

Q16 $\ddot{x} = -4x$

a) $x(0) = 0$

$v(0) = 4$

Let $x = A\sin(2t)$

$v = 2A\cos(2t)$

$4 = 2A$

$\underline{A = 2}$

$\therefore x = 2\sin(2t)$

b) $x(0) = 4$

$v(0) = 0$

Let $x = A\cos(2t)$

$\underline{x = 4\cos(2t)}$

Q17. $\ddot{x} = -64x$

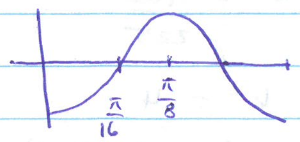
a) $|a| = 2\text{cm}$

b) $k = 8$

$\therefore P = \frac{2\pi}{8}$

$= \frac{\pi}{4}$ secs.

c)



$\therefore t = \frac{\pi}{16}$ secs

d) $x = -2\cos(8t)$

$v = 16\sin(8t)$

$|v(\frac{\pi}{16})| = 16\sin(\frac{8\pi}{16})$

$= 16$ cm/s

e) $\max |v| = 16$

$\therefore |v| = 8$

$\Rightarrow v = \pm 8$

$\pm \frac{1}{2} = \sin(8t)$

$8t = \frac{\pi}{6}$

$t = \frac{\pi}{48}$ secs

Q18. $x = -4\sqrt{3}\sin(2t) - 4\cos(2t)$

a) $x(0) = -4\sqrt{3}(0) - 4(1)$

$= -4$

$\therefore$ 4 m

b) $x = -4\sqrt{3}\sin(2t) - 4\cos(2t)$

$v = -8\sqrt{3}\cos(2t) + 8\sin(2t)$

$a = 16\sqrt{3}\sin(2t) + 16\cos(2t)$

$\ddot{x} = -4x \quad \therefore k = 2$

$\therefore$ simple harmonic motion.

c) $D = \int_0^{1.5} |v(t)|\,dt$

$D = 14.98$ m

NOTE :

$x = A\sin(kt + \alpha)$

$A = \sqrt{16(3) + 16}$

$= \sqrt{64}$

$A = 8$

$x = 8\sin(2t + \alpha)$

$\alpha = \tan^{-1}\left(\frac{-4}{-4\sqrt{3}}\right)$

$= -\frac{5\pi}{6}$

$\therefore x = 8\sin(2t - \frac{5\pi}{6})$

Q19. $x = 3 + 4\sin(\pi t)$

$x - 3 = 4\sin(\pi t)$

$p = 4\sin(\pi t)$

$\dot{p} = 4\pi\cos(\pi t)$

$\ddot{p} = -4\pi^2\sin(\pi t)$

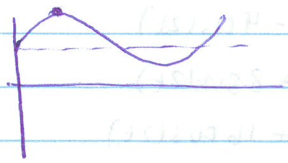
$\ddot{p} = -\pi^2 p$, $k = \pi$

b) Given simple harmonic motion,

$|a| = 4m$, $P = 2$ secs

c) $B \Rightarrow 3m$

d)



7m

Q20. $x = 5 - 3\cos(2t)$

$x - 5 = -3\cos(2t)$

$s = -3\cos(2t)$

$\dot{s} = 6\sin(2t)$

$\ddot{s} = 12\cos(2t)$

$\ddot{s} = -4s$, $k = 2$

b) Given simple harmonic motion,

$|a| = 3$, $P = \frac{2\pi}{2} = \pi$ secs

c) $P = 5m$.

d) $5 - 3 = 2m$

Q21 $v = \frac{1}{4}\cos(t)$

a) $\int_0^1 \left|\frac{1}{4}\cos(t)\right| dt$

$= 0.2104$

$\approx 0.21m$

b) $\int_0^2 \left|\frac{1}{4}\cos(t)\right| dt$

$= 0.2727$

$\approx 0.27m$

Q22. $v^2 = k^2(A^2 - x^2)$

$900 = k^2(A^2 - 400)$ (1)

$196 = k^2(A^2 - 576)$ (2)

$\therefore \frac{900}{A^2 - 400} = \frac{196}{A^2 - 576}$

$900(A^2 - 576) = 196(A^2 - 400)$

$900A^2 - 196A^2 = 440000$

$704A^2 = 440000$

$A^2 = 625$

$|A| = 25m$

$900 = k^2(225)$

$k^2 = \frac{900}{225}$

$k^2 = 4$

$\therefore k = 2 \Rightarrow P = \pi$ secs

Q23. $v^2 = k^2(A^2 - x^2)$

$0.75^2 = k^2(A^2 - 0.6^2)$ (1)

$1.56^2 = k^2(A^2 - 0.39^2)$ (2)

Solving on CAS,

$\therefore |A| = 0.65m$, $|k| = 3$

$\therefore P = \frac{2\pi}{3}$ secs